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Robust Variance-Constrained Control for a Class Continuous Markov Jump Systems

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Abstract

For a class continuous Markov jump systems, the variance-constrained and uncertain robust performance problem is researched, which guarantees the closed-loop steady-state variance to be less than a given upper, and the conditions for the existence of such controllers are proposed and proved. The designed method of these controllers is proposed. The simulation results show that the designed controller meets the demands of stability, robustness and variance-constrained.

Keywords: performance, variance-constrained, Markov jump systems, Linear matrix inequality

1. Introduction

In practical engineering applications , for the existence of random mutation in system model , the system structure often changes , such as the mutation of environmental conditions、the changing of the parameters and so on . researches found that these random changes often follow the changing law of Markovian process , markov jump system can describe this system , essay[1-8] do some research about this system , and have got some conclusions , but never researched , for a class continuous Markov jump systems , the variance-constrained and uncertain robust performance problem is researched.

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2. Problem Description

In given complete probability space, consider the following MARKOV jumping system with parameters uncertain:

$$\begin{cases} \dot{x}_t = (A(r_t) + \Delta A(t, r_t))x_t + B(r_t)u_t + G(r_t)w_t, x(0) = x_0 \\ z_t = C(r_t)x_t + D(r_t)u_t + L(r_t)w_t \end{cases} \tag{1}$$

Where state $x_2 \in R^{n_x}$,and satisfy $E[x_0] = 0$,control input $u_2 \in R^{n_x}$,control output $z_t \in R^{n_x}$, w_t is the white noise ,and it satisfies:

$E\{dw_t\} = 0$, $E\{dw_t^2\} = Wdt$, $W > 0$

r_t is a continuous-time discrete-state Markov process with values in finite set with transition probability matrix :

$$\Pr\{r_{t+\Delta} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta) & i \neq j \\ 1 + \pi_{ii}\Delta + o(\Delta) & i = j \end{cases} \tag{2}$$

In this relation $\Delta > 0$, π_{ij} is the transition rate from mode i to mode j , for all $i, j \in \mathbb{S}$,it satisfies

$\pi_{ij} \geq 0 (i \neq j)$ and $\pi_{ii} = -\sum_{j=1, j\neq i}^{s} \pi_{ij}$.

For each $r_t = i \in \mathbb{S}$,uncertain parameters fulfil matched condition:

$$\Delta A(t, i) = H_i F_t(i) M_i \tag{3}$$

Where real matrix $F_t(i)$ represent the uncertainty of parameters, it fulfills:

$$F_t^T(i) F_t(i) \leq I \quad \forall i \in \mathbb{S} \tag{4}$$

Suppose to system (1), the feedback control law can be as follows:

$$u_t = K(r_t)x_t \tag{5}$$

Then system (1) can be written in the following compact form:

$$\begin{cases} \dot{x}_t = \overline{A}(r_t)x_t + G(r_t)w_t \\ z_t = \overline{C}(r_t)x_t + L(r_t)w_t \end{cases} \tag{6}$$

Where $\overline{A}(r_t) = A(r_t) + B(r_t)K(r_t) + H(r_t)F_t(r_t)M(r_t)$, $\overline{C}(r_t) = C(r_t) + D(r_t)K(r_t)$

For each static $r_t \in \mathbb{S}$, $A(r_t)$, $B(r_t)$, $G(r_t)$, $C(r_t)$, $D(r_t)$, $H(r_t)$ is a constant interconnection matrix with proper dimension. For $r_t = i \in \mathbb{S}$,assume $[\cdot](r_t = i) = [\cdot]_i$ 。

Definition 1: For each $i \in \mathbb{S}$,define:

$$\hat{Q}_t(i) := E[x_t(i)x_t(i)^T] = E[x_t(r_t)x_t(r_t)^T \mid r_t = i]$$
$$\hat{Q}(i) := \lim_{t\to\infty} E[x_t(i)x_t(i)^T] = \lim_{t\to\infty}\hat{Q}_t(i)$$

Then:

$$\hat{Q}_t(r_t) = E[x_t(r_t)x_t(r_t)^T] = \sum_{i=1}^{s} E[x_t(i)x_t(i)^T \mid r_t = i]P_r\{r_t = i\}$$

If system (1) is robust stochastic stability , then steady-state covariance of the state exist ,that is

$$\hat{Q}(r_t) = \lim_{t\to\infty}\hat{Q}_t(r_t) = \lim_{t\to\infty}\sum_{i=1}^{s}\hat{Q}_t(i)P_r\{r_t = i\} = \lim_{t\to\infty}\sum_{i=1}^{s}\hat{Q}_t(i)P_r = \sum_{i=1}^{s}\hat{Q}(i)P_r$$

The aim of this paper is to design a state feedback controller of system (1) as equation (5) for all the allowable uncertainties, it fulfill the following three requirements:

(P1) System (1) is stochastic stability;

(P2) System (1) is robustness； For all $r_0 \in \mathbb{S}$,there exists a constant $M(x_0, r_0)$ which satisfy $M(0, r_0) = 0$,to given scalar $\gamma > 0$, the initial state $x_0 \in \mathbb{R}^{n_x}$ ， $r_0 \in \mathbb{S}$ ， the controlled output $z(t)$ satisfy :

$$\left[E\int_0^T z^T(t)z(t)dt \mid (x_0, r_0)\right]^{\frac{1}{2}} \le \gamma\left[\|w\|_2^2 + M(x_0, r_0)\right]^{\frac{1}{2}} \tag{7}$$

(P3) The stable variances for each state meet the following constraints:

$$Var[x_t(r_t)] := \lim_{t\to\infty} E[x_{t,i}(r_t)x_{t,i}^T(r_t)] = [\hat{Q}(r_t)]_{ii} < \sigma_i^2(r_t) \tag{8}$$

Where $x_t(r_t) = [x_{t,1}(r_t) \ \cdots \ x_{t,n_x}(r_t)]^T$,$[\hat{Q}(r_t)]_{ii}$ is the elements on the diagonal matrix of $[\hat{Q}(r_t)]$,the given scalar $\sigma_i^2(r_t) > 0 \ \ (i = 1,\cdots,n_x)$ is the acceptable upper bound variance required by practical problems.

3. Control Design

This section will deduce a multi targets controller design for continuous Markov jump system with parameters uncertain.

Theorem 1 To the a given positive constant $\gamma > 0$ ， $\sigma_{ij}^2 > 0 \ \ (i = 1,\cdots,s, j = 1,\cdots,n_x)$,if there exist constant $\varepsilon_i > 0$, $\zeta_i > 0$ the symmetric positive definite matrix $X_i > 0$ and Y_i :

$$\begin{bmatrix} A_iX_i + X_iA_i^T + B_iY_i + Y_i^TB_i^T + \pi_{ii}X_i + \zeta_iH_iH_i^T & G_i & X_iC_i^T + Y_i^TD_i^T & X_iM_i^T & \mathrm{S}_i(X) \\ G_i^T & -\gamma^2 I & L_i^T & 0 & 0 \\ C_iX_i + D_iY_i & L_i & -I & 0 & 0 \\ M_iX_i & 0 & 0 & -\zeta_i I & 0 \\ \mathrm{S}_i^T(X) & 0 & 0 & 0 & -\mathrm{X}_i(X) \end{bmatrix} < 0 \tag{9}$$

$$\begin{bmatrix} A_iX_i + X_iA_i^T + B_iY_i + Y_i^TB_i^T + \pi_{ii}X_i + \varepsilon_iH_iH_i^T + \sum_{j=1}^{s}\pi_{ij}X_j + G_iWG_i^T & X_iM_i^T \\ M_iX_i & -\varepsilon_iI \end{bmatrix} < 0 \tag{10}$$

$$\left[X_i\right]_{jj} \le \sigma_{ij}^2, \quad (i = 1,\cdots,s, j = 1,\cdots,n_x) \tag{11}$$

Then there exists a feedback controller like equation (5) satisfy all the acceptable uncertainties in （P1）~（P3）, and the feedback control gain is $K_i = Y_iX_i^{-1}$.

Proof:(1) First prove there exist a group of symmetric positive definite matrix $X_i > 0$, which satisfy the following constraints:

$$\overline{A}_iX_i + X_i\overline{A}_i^T + \sum_{j=1}^{s}\pi_{ij}X_j + G_iWG_i^T \le 0, \qquad i \in \mathbb{S} \tag{12}$$

Then the closed loop system is stochastic stable.

$$\rho(\overline{A}) = \rho(\overline{A}^T)$$

Where $\rho(\cdot)$ is the spectral radius of a matrix.So when $u_t \equiv 0$,the stability requirements for the following system are equal to system (1):

$$dx_t = \overline{A}^T(r_t)dx_t + G(r_t)dw_t \tag{13}$$

Construct a Lyapunov function:

$$V(x_t, r_t = i) = V(x_t, i) = x_t^TX_ix_t, \quad X_i > 0, \quad i \in \mathbb{S}$$

By weak infinitesimal generator there is

$$\mathrm{L}V(x_t, i) = x_t^T(\overline{A}_iX_i + X_i\overline{A}_i^T + \sum_{j=1}^{s}\pi_{ij}X_j + G_iWG_i^T)x_t$$

Notes that $G_iWG_i^T > 0$, obviously ,for close loop system ,if there exist one group of symmetric positive definite matrix $X_i > 0$,when equation(12) is true ,then

$$\mathrm{L}V(x_t, i) = x_t^T(\overline{A}_iX_i + X_i\overline{A}_i^T + \sum_{j=1}^{s}\pi_{ij}X_j)x_t < 0$$

There exist one group of symmetric positive definite matrix $Q_i > 0$, $i \in \mathbb{S}$,which is assumed to satisfy a constraint:

$$\mathrm{L}V(x_t, i) = x_t^T(\overline{A}_iX_i + X_i\overline{A}_i^T + \sum_{j=1}^{s}\pi_{ij}X_j)x_t = -x_t^TQ_ix_t$$

Then

$$\mathrm{L}V(x_t, i) \le -\min_{i\in\mathbb{S}}\lambda_{\min}[Q_i]x_t^Tx_t$$

By Dynkin lemma, there is

$$\mathbb{E}[V(x_t, i)] - V(x_0, r_0) = E\left[\int_0^t \mathrm{L}V(x_s, r_s)ds\right] \le -\min_{i\in\mathcal{S}}\lambda_{\min}[Q_i]E\left[\int_0^t x_s^Tx_sds \mid (x_0, r_0)\right]$$

From this equation , there is

$$\min_{i\in\mathcal{S}}\lambda_{\min}[Q_i]E\left[\int_0^t x_s^Tx_sds \mid (x_0, r_0)\right] \le E[V(x_0, r_0)] - E[V(x_t, i)] \le E[V(x_0, r_0)]$$

So $E\left[\int_0^t x_s^T x_s ds \mid (x_0, r_0)\right] \le \dfrac{E[V(x_0, r_0)]}{\min_{i\in\mathbb{S}} \lambda_{\min}[Q_i]}$

Define $T(x_0, r_0) = \dfrac{E[V(x_0, r_0)]}{\min_{i\in\mathbb{S}} \lambda_{\min}[Q_i]}$,when $t \to \infty$,there is

$$E\left[\int_0^\infty x_s^T x_s ds \mid (x_0, r_0)\right] \le T(x_0, r_0)$$

From essay [9] ,system which meet the condition of equation (12) is random stable.

(2) Prove the steady-state covariance matrix of closed-loop system $\hat{Q}_i$ satisfy $\hat{Q}_i \le X_i$.

By definition 1, the covariance of the system can be defined as

$$\hat{Q}_t(i) := E[x_t(i)x_t(i)^T] = E[x_t(r_t)x_t(r_t)^T \mid r_t = i]$$

By weak infinitesimal generator

$$\mathrm{L}\hat{Q}_t(i) = \overline{A}_i\hat{Q}_t(i) + \hat{Q}_t(i)\overline{A}_i^T + \sum_{j=1}^{s}\pi_{ij}\hat{Q}_t(j) + G_iWG_i^T$$

When $t \to \infty$,the steady state variance of the system satisfies:

$$\lim_{t\to\infty}\mathrm{L}\hat{Q}_t(i) = \overline{A}_i\hat{Q}_i + \hat{Q}_i\overline{A}_i^T + \sum_{j=1}^{s}\pi_{ij}\hat{Q}_j + G_iWG_i^T = 0 \tag{14}$$

By using Equation (14) to minus (16) , there is

$$\overline{A}_i[X_i - \hat{Q}_i] + [X_i - \hat{Q}_i]\overline{A}_i^T + \sum_{j=1}^{s}\pi_{ij}[X_j - \hat{Q}_j] \le 0$$

From essay [10] there is, $\hat{Q}_i \le X_i$.

(3) The design of system which is random mean-square stability and satisfy all the upper bound. Proof (1)、(2) shows when AS satisfy equation (12) ,system is random mean-square stability and the steady covariance matrix of system state satisfy all the upper bound, then we discuss the design of system which is random mean-square stability and satisfy all the upper bound when satisfying equation (10), (11).

To facilitate processing, define

$$\begin{cases}\mathrm{S}_i(X) = \left[\sqrt{\pi_{i1}}X_i \quad \cdots \quad \sqrt{\pi_{ii-1}}X_i \quad \sqrt{\pi_{ii+1}}X_i \quad \cdots \quad \sqrt{\pi_{is}}X_i\right] \\ \mathrm{X}_i(X) = diag\left[X_1, \cdots, X_{i-1}, X_{i+1}, \cdots, X_s\right]\end{cases} \tag{15}$$

So

$$X_i\left[\sum_{j=1}^{s}\pi_{ij}X_j^{-1}\right]X_i = \pi_{ii}X_i + \mathrm{S}_i(X)\mathrm{X}_i^{-1}(X)\mathrm{S}_i^T(X) \tag{16}$$

Made $\overline{A}_i = A_i + B_iK_i + H_iF_t(i)M_i$ in(12),then

$$A_iX_i + X_iA_i^T + B_iK_iX_i + X_iK_i^TB_i^T + H_iF_t(i)M_iX_i + X_i(H_iF_t(i)M_i)^T + \sum_{j=1}^{s}\pi_{ij}X_j + G_iWG_i^T \le 0$$

From easy [11] ,there exist a positive constant $\varepsilon > 0$,which makes:

$$H_iF_t(i)M_iX_i + X_i(H_iF_t(i)M_i)^T \le \varepsilon_i H_i H_i^T + \varepsilon_i^{-1} X_i M_i^T M_i X_i \tag{17}$$

Then

$$A_iX_i + X_iA_i^T + B_iK_iX_i + X_iK_i^TB_i^T + H_iF_t(i)M_iX_i + X_i(H_iF_t(i)M_i)^T + \sum_{j=1}^{s}\pi_{ij}X_j + G_iWG_i^T$$

$$\le A_iX_i + X_iA_i^T + B_iK_iX_i + X_iK_i^TB_i^T + \varepsilon_iH_iH_i^T + \varepsilon_i^{-1}X_iM_i^TM_iX_i + \sum_{j=1}^{s}\pi_{ij}X_j + G_iWG_i^T \le 0$$

Let $Y_i = K_iX_i$,then

$$A_iX_i + X_iA_i^T + B_iY_i + Y_i^TB_i^T + \varepsilon_iH_{1i}H_{1i}^T + \varepsilon_i^{-1}X_iM_i^TM_iX_i + \sum_{j=1}^{s}\pi_{ij}X_j + G_iWG_i^T \le 0$$

By Schurz, we can get equation (10).

From proof (1)、(2), we know when equation (10) is true , the system is random mean-square stability and the steady covariance matrix of system state satisfy $\hat{Q}_i \le X_i$ .Consider the steady state variance constraint equation of the system state (11)，There is the following conclusion:

If there exist a constant $\varepsilon > 0$ and symmetric positive definite matrix $X_i > 0$,matrix Y_i, makes matrix inequality (12)、(13)true. then state feedback controller (6) with gain $K_i = Y_iX_i^{-1}$ make closed loop system (7)meet the performance of (P1) and (P3).

(4) Prove system (7) is random robustness stable

Introducing matrix inequality

$$\begin{bmatrix} \bar{A}_i^TP_i + P_i\bar{A}_i + \sum_{j=1}^{s}\pi_{ij}P_j + \bar{C}_i^T\bar{C}_i & \bar{C}_i^TL_i + P_iG_i \\ L_i^T\bar{C}_i + G_i^TP_i & L_i^TL_i - \gamma^2 I \end{bmatrix} < 0 \tag{18}$$

Where the positive constant $\gamma > 0$, positive definite matrix $P_i > 0$, $i \in \mathbb{S}$,Equation(18)can be written by

$$\begin{bmatrix} \bar{A}_i^TP_i + P_i\bar{A}_i + \sum_{j=1}^{s}\pi_{ij}P_j + \bar{C}_i^T\bar{C}_i & \bar{C}_i^TL_i + P_iG_i \\ L_i^T\bar{C}_i + G_i^TP_i & L_i^TL_i - \gamma^2 I \end{bmatrix} = \begin{bmatrix} \bar{A}_i^TP_i + P_i\bar{A}_i + \sum_{j=1}^{s}\pi_{ij}P_j & P_iG_i \\ G_i^TP_i & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \bar{C}_i^T \\ L_i^T \end{bmatrix}\begin{bmatrix} \bar{C}_i & L_i^T \end{bmatrix} < 0$$

From Schur fill lemma , the equation can be described as the following inequality

$$\begin{bmatrix} \bar{A}_i^TP_i + P_i\bar{A}_i + \sum_{j=1}^{s}\pi_{ij}P_j & P_iG_i & \bar{C}_i^T \\ G_i^TP_i & -\gamma^2 I & L_i^T \\ \bar{C}_i & L_i & -I \end{bmatrix} \langle\, 0 \tag{19}$$

Where $\bar{A}_i = A_i + B_iK_i + H_iF_t(i)M_i$, $\bar{C}_i = C_i + D_iK_i$.

Notice equation (21)is NLMI about K_i and P_i ,facilitate solving ,convert it to LMI, let $X_i = P_i^{-1}$,and premultiply and post multiply $diag[X_i, I, I]$ to equation (21),there is

$$\begin{bmatrix} X_i\bar{A}_i^T+\bar{A}_iX_i+\sum_{j=1}^{s}\pi_{ij}X_iX_j^{-1}X_i & G_i & X_i\bar{C}_i^T \\ G_i^T & -\gamma^2 I & L_i^T \\ \bar{C}_iX_i & L_i & -I \end{bmatrix}<0$$

Let $Y_i = K_iX_i$,introduced equation(17),notice

$$X_i\bar{A}_i^T+\bar{A}_iX_i = X_iA_i^T+A_iX_i+Y_i^TB_i^T+B_iY_i+H_iF_t(i)M_iX_i+X_i(H_iF_t(i)M_i)^T$$

$$X_i\left[\sum_{j=1}^{s}\pi_{ij}X_j^{-1}\right]X_i=\pi_{ii}X_i+\mathrm{S}_i(X)\mathrm{X}_i^{-1}(X)\mathrm{S}_i^T(X)$$

$$X_i(C_i+D_iK_i)^T=X_iC_i^T+Y_i^TD_i^T$$

From essay[11],we know there exists a positive constan $\zeta_i>0$,which makes

$$H_iF_t(i)M_iX_i+X_i(H_iF_t(i)M_i)^T\le\zeta_iH_iH_i^T+\zeta_i^{-1}X_iM_i^TM_iX_i$$

From Schur fill lemma, we can get equation (11).
From above we know equation(11)is equal to equation(20)
Form Schur fill lemma , when equation(20)is true, there is

$$\bar{A}_i^TP_i+P_i\bar{A}_i+\sum_{j=1}^{s}\pi_{ij}P_j+\bar{C}_i^T\bar{C}_i<0$$

For every $i\in\mathbb{S}$, $\bar{C}_i^T\bar{C}_i\ge 0$,there is

$$\bar{A}_i^TP_i+P_i\bar{A}_i+\sum_{j=1}^{s}\pi_{ij}P_j<0$$

From essay[9],we know system(1)、(2) is random stable
Next we prove equation(20) makes system have the disturbance attenuation γ
Take Lyapunov function

$$V(x_t,r_t=i)=V(x_t,i)=x_t^TP_ix_t\ ,\ P_i>0\ ,\ i\in\mathbb{S}$$

By weak infinitesimal generator there is

$$\mathrm{L}V(x_t,i)=x_i^T(\bar{A}_i^TP_i+P_i\bar{A}^T+\sum_{j=1}^{s}\pi_{ij}P_j)x_t+2x_t^TP_iG_iw_i$$

Define the indicator function

$$J_T=E\left[\int_0^T(z_t^Tz_t-\gamma^2w_t^Tw_t)dt\right]$$

Notice

$$\begin{aligned} &z_t^Tz_t-\gamma^2w_t^Tw_t+\mathrm{L}V(x_t,i) \\ &=x_t^T\bar{C}_i^T\bar{C}_ix_t-\gamma^2w_t^Tw_t+x_t^T\bar{C}_i^TL_iw_t+w_t^TL_i^T\bar{C}_ix_t+w_t^TL_i^TL_iw_t \quad + \\ &x_i^T(\bar{A}_i^TP_i+P_i\bar{A}^T+\sum_{j=1}^{s}\pi_{ij}P_j)x_t+x_t^TP_iG_iw_i+w_t^tG_i^TP_ix_t \\ &=\begin{bmatrix} x_t^T & w_t^T \end{bmatrix}\begin{bmatrix} \bar{A}_i^TP_i+P_i\bar{A}_i+\sum_{j=1}^{s}\pi_{ij}P_j+\bar{C}_i^T\bar{C}_i & \bar{C}_i^TL_i+P_iG_i \\ L_i^T\bar{C}_i+G_i^TP_i & L_i^TL_i-\gamma^2 I \end{bmatrix}\begin{bmatrix} x_t \\ w_t \end{bmatrix} \end{aligned}$$

There is

$$J_T = E\left[\int_0^T (z_t^T z_t - \gamma^2 w_t^T w_t + \mathrm{L}V(x_t,i))dt\right] - E\int_0^T \mathrm{L}V(x_t,i)dt$$

From Dynkin fill lemma , there is

$$E\int_0^T \mathrm{L}V(x_t,i)dt = E[V(x_T,r_T)] - V(x_0,r_0)$$

There is

$$J_T = E\int_0^T \begin{bmatrix} x_t^T & w_t^T \end{bmatrix} \begin{bmatrix} \bar{A}_i^T P_i + P_i\bar{A}_i + \sum_{j=1}^{s}\pi_{ij}P_j + \bar{C}_i^T\bar{C}_i & \bar{C}_i^T L_i + P_i G_i \\ L_i^T\bar{C}_i + G_i^T P_i & L_i^T L_i - \gamma^2 I \end{bmatrix} \begin{bmatrix} x_t \\ w_t \end{bmatrix} dt$$
$$- E[V(x_T,r_T)] + V(x_0,r_0)$$

From $E[V(x_T,r_T)] \ge 0$ and equation (20),there is

$$J_T \le V(x_0,r_0)$$

When $T \to \infty$,there is

$$\|z\|_2 \le \gamma\left[\|w\|_2^2 + x_0^T P(r_0)x_0)\right]^{\frac{1}{2}}$$

From aboved proof ,we know when equation(11) is true , Then the closed loop system 6 is stochastic stable and have the disturbance attenuation γ , which fulfills (P2)

4. Simulation Example

In order to compare easily, take the same model like essay[9].the parameters in system (7) is as follows:

Model transfer rate matrix: $\Pi = \begin{bmatrix} -2.0 & 2.0 \\ 3.0 & -3.0 \end{bmatrix}$

Model 1($r_t = 1$):

$A(1) = \begin{bmatrix} 1 & -0.5 \\ 0.1 & 1.0 \end{bmatrix}$, $B(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $G(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $L(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $D(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

$M(1) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $H(1) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$.

Model 2($r_t = 2$):

$A(2) = \begin{bmatrix} -0.2 & -0.5 \\ 0.5 & -0.25 \end{bmatrix}$, $B(2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $G(2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $L(2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C(2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

$D(2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $M(2) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $H(2) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $W = 1$.

Find a right key to theorem 1,Let $\gamma = 1$. $\sigma_{ij}^2 = 1$ ($i = 1,2$, $j \in 1,2$).Use Matlab LMI function box to solve equation (11)~(13), there is

$X_1=\begin{bmatrix}23.6753 & 3.2561\\ 3.2561 & 12.2635\end{bmatrix}$, $X_2=\begin{bmatrix}31.3624 & -3.5614\\ -3.5614 & 23.2549\end{bmatrix}$, $Y_1=\begin{bmatrix}-39.4531 & -3.5784\\ -3.4362 & -35.4876\end{bmatrix}$,

$Y_2=\begin{bmatrix}-45.6316 & 2.5387\\ 2.5368 & -43.7025\end{bmatrix}$, $\zeta_1=43.5294$, $\zeta_2=21.3645$, $\varepsilon_1=5.3215$, $\varepsilon_2=7.1245$.

So the control gain is $K_1=\begin{bmatrix}-1.6879 & 0.1564\\ 0.2624 & -2.9634\end{bmatrix}$, $K_2=\begin{bmatrix}-1.4681 & -0.1157\\ -0.1349 & -1.8999\end{bmatrix}$

5. Conclusion

For a class continuous Markov jump systems, the variance-constrained and uncertain robust performance problem is researched. research sufficient conditions for its random mean-square stability by using LMI forward and prove the condition for robust controller with variance constraint, and the design of robust variance controller; compared with the controller designed by using algebraic method description, the controller proposed by the essay is more convenient ,and can use Matlab LMI function box to solve directly ,the simulation result shows the designed controller meet the demands of stability, robustness and variance constrained .

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